Quantum Weak Measurement and its Implementation in Optical Systems

Independent Study

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An experiment proving quantized nature of intrinsic angular momentum

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For spin half particles, only two values of intrinsic angular momentum values are possible $\pm\hbar/2$

Stern-Gerlach Experiment

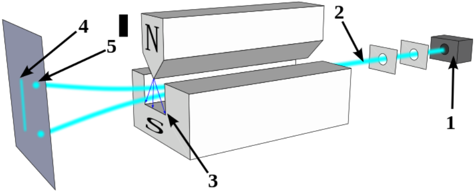

Figure: Classical expectation versus quantum result of the SG experiment

Weak Measurement?

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Weak measurement - "Not much" change in the state
What does that mean in SG?

Weak Measurement in SG

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The final state of the system after measurement is still a mixture of both the spins.

Post Selection

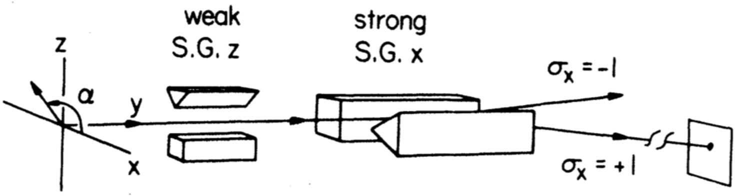

Figure: Weak measurement followed by strong measurement and post selection. Source: Duck, Sudarshan, Phys.Rev.D (1989)

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Claim: If we start out with a spin $1/2$ particle, after the above processes, we can end up with a spin 100 reading

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Coupling of the measurement device to the observable: von Neumann model
$\hat{H} = -g(t)\hat{q}\hat{A}$
where

- $g(t)$ is a function with compact support near the time of measurement and normalised
- $\hat{q}$ is the canonical variable of the measuring device and $\hat{p}$ is its conjugate momentum

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Hence $\Delta q = \Delta$
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After some math:

$$|\Phi_f\rangle = \sum_n \alpha_n \alpha'_n \int dp exp(-\Delta^2 (p - a_n)^2)\,|p\rangle \tag{3}$$

A summation of gaussians centred at the eigenvalues of $\hat{A}$

Post Selection - AAV Method

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Finally,

$$|\Phi_f\rangle \approx \langle \Psi_f | \Psi_{in} \rangle \int dp e^{-\Delta^2 (p - A_w)^2} \, |p\rangle \tag{4}$$

A single gaussian centred at A_w!

Post Selection - AAV Method

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Hidden approximations!

- $|q^n \langle\Psi_f|\hat{A}^n|\Psi_{in}\rangle| \ll |\langle\Psi_f|\Psi_{in}\rangle|, n \geq 2$
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- $|qA_w| \ll 1$

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But $q \leftrightarrow \Delta$. Hence

- $\Delta \ll min_{n=2,3,\ldots} \left| \frac{\langle \Psi_f | \hat{A} | \Psi_{in} \rangle}{\langle \Psi_f | \hat{A}^n | \Psi_{in} \rangle} \right|^{1/n-1}$
- $\Delta \ll 1/A_w$

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will have (N+1) equally spaced eigenvalues.

Weak Value

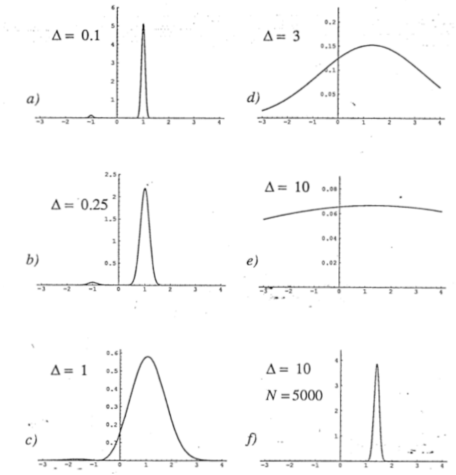

Figure: Weak value of spin. Source: Vaidman "Weak Measurement" https://arxiv.org/abs/hep-th/9408154

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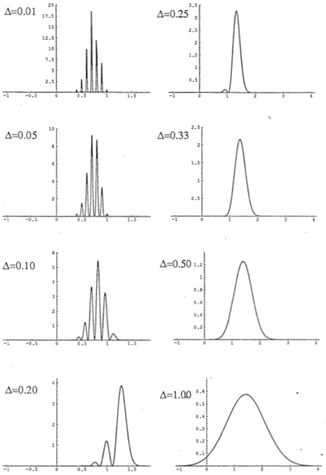

Figure: Weak value of spin of N particles. Source: Vaidman "Weak Measurement" https://arxiv.org/abs/hep-th/9408154

Back to Stern Gerlach experiment

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System: z-spin
Probe: z-momentum

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Hence the weak value of spin is:
$A_w = (\lambda\sigma_z)_w = \lambda \tan\alpha/2$

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$\Delta \ll \lambda^{-1} min[\tan \alpha/2, \cot \alpha/2]$

Playing with validity

Condition to be valid for AAV's approximation:
$\Delta \ll \lambda^{-1} min[\tan\alpha/2, \cot\alpha/2]$
Hence α must not go too close to π but it can be taken arbitrarily close to π so as to measure a spin value of 100

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The Setup

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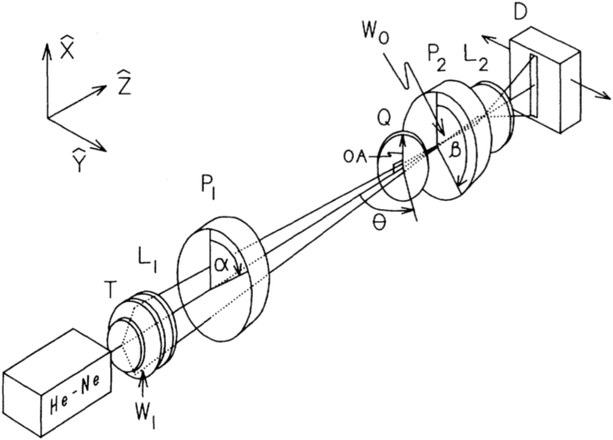

Figure: Optical version of weak measurement (Source: Ritchie et al. Phys. Rev. Lett. (1990)

The Math

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First polarization:

$$\vec{E_i} = E_0 G(x) G(y) (\cos\alpha\hat{x} + \sin\alpha\hat{y})$$

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$$\vec{E_w} = E_0 G(x) [\cos\alpha G(y+a) e^{i\phi} \hat{x} + \sin\alpha G(y) \hat{y}]$$

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$$\vec{E_f} = E_0 G(x)[\cos\alpha\cos\beta G(y+a)e^{i\phi} + \sin\alpha\sin\beta G(y)](\cos\beta\hat{x} + \sin\beta\hat{y})$$

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where I_0 is proportional to $|E_0|^2$
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- $\beta = \alpha \Rightarrow$ constructive superposition; single, unshifted Gaussian
- $\beta = \alpha + \pi/2 + \epsilon$; $(\epsilon \ll 1) \Rightarrow$ destructive interference. Weak value $A_w \approx a\cot(\epsilon)/2$

Results

Results

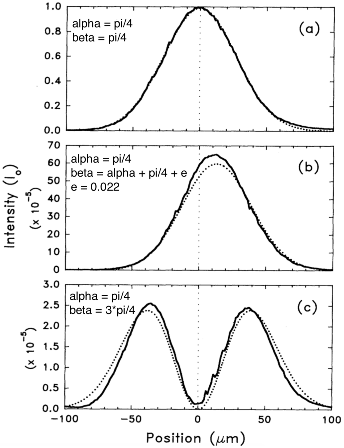

Figure: Intensity in the detector as a function of y when $a = 0.64\mu$m and $\Delta \approx 55\mu$m (Source: Ritchie et al. Phys. Rev. Lett. (1990))

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What we obtain by repeated measurement here is only the real value of A_w.
Imaginary of A_w:

- Shifts the momentum operator of the probe system.
- Determines the probability of post selection.

The imaginary component does not affect the weak value nor the average quantities.

Results from Kedem's paper

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$< Q >_N = N < Q > = Nc$ and
$\Delta_N = \sqrt{N}\Delta$
Hence,

$$R = \sqrt{N}\frac{c}{\Delta} \tag{6}$$

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where $N_{\Phi} = N|\langle\Phi|\Psi\rangle|^2$

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Standard deviation of P $= 1/\sqrt{2}\Delta$
Hence the SNR is:

$$R = \sqrt{N_\Phi}\frac{Im(C_w)}{\Delta} \tag{8}$$

Hence if $Re(C_w) = Im(C_w)$ then R in both cases would be the same.

Kedem's results

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Hence, SNR will have the same shift but with larger deviation.

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But there is a twist in the tale!

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In this case, both the signal value and the deviation have changed.

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By measuring K, we obtain information regarding just the imaginary part of the weak value.

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Initial standard deviation of shift in K is maintained! Completely mixed state will have maximum standard deviation and hence the greatest SNR!

Noise tolerance

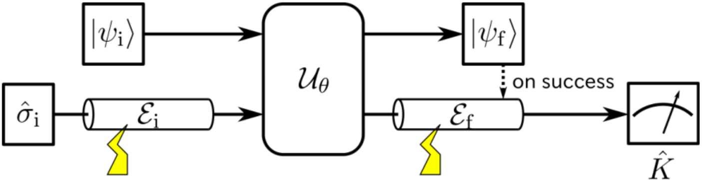

Figure: Setup for weak measurement with mixed probe states. Noise is introduced before and after probing. `http://arxiv.org/abs/1211.4292v1`

The result of the weak measurement is unaffected by the introduction of the phase noises.

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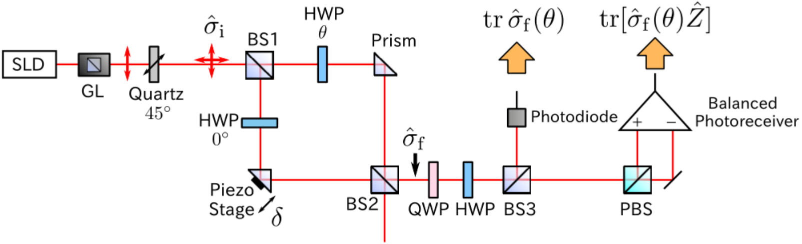
Setup



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Hence the effective evolution is: $U(\theta) = exp(2i\theta P_0 Z)$ where P_0 is the projection (to $|0\rangle\langle 0|$) operator.

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This is experimentally obtained by plotting and finding the slope near $\theta = 0$

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when V = 1, we get back the original case.

Results

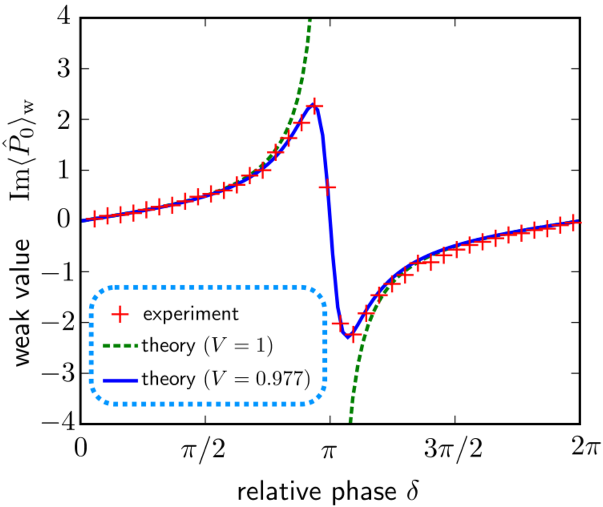

Figure: `http://arxiv.org/abs/1211.4292v1`

Thank you!

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